Fast Monte Carlo Localization of AUV Using Acoustic Range Measurement

Sajad Saeedi†, Mae Seto⋆, and Howard Li†

Abstract: This paper presents a novel online nonlinear Monte Carlo algorithm for multi-sensor autonomous underwater vehicle (AUV) navigation. The approach integrates the global constraints of range to, and GPS position of, multiple surface vehicles communicated via acoustic modems and relative pose constraints arising from observations of multiple beacon boats. The proposed method can be used to more accurately navigate the AUV, to extend mission duration, and to avoid surfacing for GPS fixes. The Monte Carlo method is used for the estimation of the AUV pose. Although it is also desirable to estimate the range measurement of the surface vehicles using a particle filter (PF), implementing a PF for each beacon onboard the AUV is computationally expensive. Thus for the range estimation, an extended Kalman filter (EKF) is proposed for each beacon. We discuss why our approach is more computationally efficient and suitable for use on underwater vehicles. Simulation results are provided for AUV navigation using multiple autonomous surface vehicles (ASVs) in an ocean environment. During these simulations the proposed algorithm runs online on-board the AUV. In-water validation is currently in progress.

Index Terms: Localization, autonomous underwater vehicle, acoustic, PF, EKF

I. Introduction

Autonomous underwater vehicles (AUVs) are now being used for a variety of tasks, including oceanographic surveys, de-mining, and bathymetric data collection in marine and riverine environments. Accurate localization and navigation is essential to ensure the accuracy of the gathered data for these applications [1].

A. Problem Statement and Literature Review

AUV navigation and localization in underwater environments is particularly challenging due to the rapid attenuation of global positioning system (GPS) and radio-frequency signals. Underwater communications are low bandwidth and unreliable, and there is no access to a global positioning system. Past approaches to solve the AUV localization problem have employed expensive inertial sensors, used boats equipped with beacons (beacon boats) in the region of interest, or required periodic surfacing of the AUV [1].

In general, AUV navigation and localization techniques can be categorized into one of three main categories [1].

- Inertial/dead reckoning: Inertial navigation uses accelerometers and gyroscopes for increased accuracy to propagate the current state. Nevertheless, all of the methods in this category have position error growth that is unbounded with time.
- Acoustic transponders and modems: Techniques in this category are based on measuring the time of flight (TOF) of signals from acoustic beacons or modems to perform navigation.
- Geophysical: Techniques that use external environmental information as references for navigation. This must be done with sensors and processing that are capable of detecting, identifying, and classifying environmental features.

In acoustic navigation techniques, localization is achieved by measuring ranges from the TOF of acoustic signals. Common methods include [1]:

- Ultra short baseline (USBL): The transducers on the transceiver are closely spaced with the approximated baseline on the order of less than 10 cm. Relative ranges are calculated based on the TOF and the bearing is calculated based on the difference of the phase of the signal arriving at the transceivers.
- Short baseline (SBL): Beacons are placed at opposite ends of a ship's hull. The baseline is based on the size of the support ship.
- Long baseline (LBL) and GPS intelligent buoys (GIBs): Beacons are placed over a wide mission area. Localization is based on triangulation of acoustic signals. In the case of GIBs, the beacons are at the surface, whereas for LBL they are installed on the seabed.
- Single fixed beacon: Localization is performed from only one fixed beacon.
- Acoustic modem: The recent advances with acoustic modems have allowed for new techniques to be developed. Beacons no longer have to be stationary, and full AUV autonomy can be achieved with support from autonomous surface vehicles, equipped with acoustic modems, or by communicating and ranging in underwater teams.

AUVs can navigate with the help of manned surface vehicles and acoustic modems. It has been demonstrated that manned surface vehicles can be replaced by unmanned surface vehicles. The first known implementation of autonomous surface vehicles (ASVs) used to support AUVs is presented in [2]. In [3], two ASVs are used and a general framework is developed for cooperative navigation. In [4], the experimental validation is done using an ASV and an AUV to compare the performance of the extended Kalman filter (EKF), particle

†COBRA Group at the University of New Brunswick, Fredericton, Canada, http://www.ece.unb.ca/COBRA/, {sajad.saeedi.g, howard}@unb.ca
⋆Defence Research and Development Canada-Atlantic, Nova Scotia, Canada, mae.seto@drdc-rddc.gc.ca

filter (PF), and a nonlinear least square (NLS) optimization. It is shown that the NLS performs the best, particularly after offline post-processing. In [5] the derivation, simulation, and experimental evaluation of a decentralized extended information filter (DEIF) algorithm for single-beacon cooperative acoustic navigation are presented. In [6] an optimized path planning for an unmanned surface vehicle (USV) is presented. The USV serves as a communication and navigation aid for multiple unmanned underwater vehicles (UUV) performing surveys.

B. Proposed Method

In this work, a fast online Monte Carlo algorithm is proposed for AUV localization using acoustic modems and a series of autonomous surface crafts equipped with acoustic beacons and GPS receivers, as shown in Fig. 1. The proposed method has two steps which are based on PF and EKF. In order to address the nonlinearity of the AUV motion, it is desirable to use the Monte Carlo method, also known as the particle filter, for the estimation of the AUV pose (position and orientation). Although it is also desirable to estimate the range measurement using a PF, implementing a PF for each beacon boat requires significant computational power which is not possible on the AUV. Thus for the range estimation, an EKF is implemented for each beacon. The proposed method is described below.

First, the Monte Carlo method is used for estimating the pose of the AUV. The prediction update of the AUV pose at time t, x_t, is realized through the kinematic model of the AUV [1]. The orientation of the AUV and the control signals at time $t-1$, u_{t-1}, are the input to the kinematic model. The importance sampling is realized through the acoustic range measurement of the beacon boats which will be described in Section II.

Then, the EKF is used to estimate the range measurement of each beacon in order to reduce the range measurement uncertainty. The predicted range measurement $\bar{\mu}_t$ is calculated using the known beacon locations m and the estimated AUV pose x_t (notice that the bar sign denotes the predicated value of a signal). The variance of the range measurement is reduced through the measurement update. The measurement update is realized using the actual acoustic range measurement y_t. Finally, to select more favoured particles that represent both the pose x_t and corresponding range estimate μ_t, importance sampling is performed.

C. Contribution

A novel AUV localization method using only acoustic range measurements is proposed. The proposed method has the advantage that it does not need too many particles to approximate the range of the beacons and the pose of the AUV. With respect to the number of the beacons, N, the complexity of the proposed algorithm is of order $\mathcal{O}(Mlog(N))$, where M is the number of the particles. For a plain EKF implementation, with a large multivariate state vector, the complexity is $\mathcal{O}(N^2)$ [7]. The proposed algorithm offers computational advantages over straightforward and regular implementation of a particle filter or an EKF. Additionally, the proposed method estimates the full trajectory of the AUV.

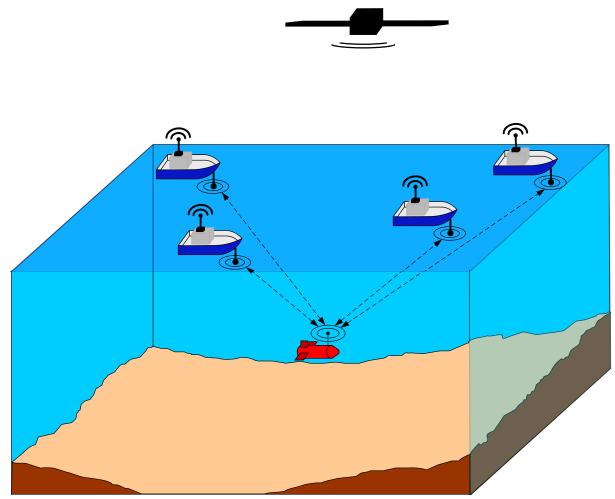

Fig. 1. Localization using ASV beacons.

D. Outline

The following section provides background information and details the proposed approach for AUV localization using multiple beacon boats. Section III describes the setup used for simulations and provides the results. Finally, the conclusion is given in Section IV.

II. Proposed Method

The proposed method is based on Bayesian filtering. Bayesian filtering, which is based on the Bayes theorem, is the core of statistical estimation and has many applications. In this section, first the Bayes filter is explained. Then the implementation of the Bayes filter using the extended Kalman filter and particle filter are explained. At the end of this section, the proposed AUV localization method is described.

A. Bayes Filter

Suppose $x_{1:t}$ represents the state of a system from time 1 to time t, $u_{1:t}$ represents all control actions from time 1 to time t, and $z_{1:t}$ represents all observations from time 1 to time t. Also x_t, z_t, and u_t represent the state of the system, observation, and control signal at time t, respectively. Often in the literature the following notations are used for simplicity [7]

$$\overline{\mathrm{bel}}(x_t) = p(x_t|z_{1:t-1}, u_{1:t}) \quad (1)$$
$$\mathrm{bel}(x_t) = p(x_t|z_{1:t}, u_{1:t}), \quad (2)$$

where the probabilities are represented by a belief function. The belief in the prior is shown by $\overline{\mathrm{bel}}(x_t)$. This probability represents the probability of x_t before incorporating the latest available measurement, z_t. The belief in the posterior is shown by $\mathrm{bel}(x_t)$. In other words, this probability represents the probability of x_t given the latest measurement, z_t. Probabilities in equations (1) and (2) are presented as [7]:

$$\overline{\mathrm{bel}}(x_t) = \int p(x_t|x_{t-1}, u_t)\ \mathrm{bel}(x_{t-1})\, dx_{t-1} \quad (3)$$

$$\mathrm{bel}(x_t) = \eta p(z_t|x_t)\overline{\mathrm{bel}}(x_t), \quad (4)$$

where in equation (4), η is a normalizer. Equations (3) and (4) construct the recursive Bayes filter. Note that the integration in equation (3) is performed over all possible values of state x_{t-1}. There is no closed form solution for this integration; therefore, to implement the Bayes filter, other approaches such the extended Kalman filter or particle filter are used.

The Kalman filter is the best known implementation of the Bayes filter for linear systems, assuming additive Gaussian noise for motion and measurement models. The Kalman filter was first proposed in 1960 by Rudolf E. Kálmán. Since then it has had many applications in engineering and estimation problems [7].

For nonlinear systems, the Kalman filter cannot be used directly. In order to apply the Kalman filter, the nonlinear equations must be linearized. This suboptimal solution is called the extended Kalman filter (EKF) and is explained in the next section.

B. Extended Kalman Filter

Assume the motion and measurement models of a nonlinear system are represented as follows:

$$x_t = f(x_{t-1}, u_t) + \delta_t \quad (5)$$

$$z_t = h(x_t) + \epsilon_t, \quad (6)$$

where $f(\cdot)$ is the nonlinear motion model, and $h(\cdot)$ is the nonlinear measurement model. δ_t is the additive process noise which is a Gaussian noise with a mean of 0 and covariance of Q. ϵ_t is the additive measurement noise which is a Gaussian noise with a mean of 0 and covariance of R. The extended Kalman filter assumes a Gaussian distribution for the state, x_t, and estimates mean and covariance of the state at time t, *i.e.*, μ_t and Σ_t. This is shown by $\mathcal{N}(x_t; \mu_t, \Sigma_t)$.

Algorithm 1 represents the EKF. In line 1, the mean of the state is predicted using the nonlinear motion model, given the control input at time t. In line 2, the covariance of the predicted state is calculated (the superscript ^{T} denotes the transpose operation). Note that F_t is the Jacobian of the motion model defined as

$$F_t = \frac{\partial f(x_{t-1}, u_t)}{\partial x_{t-1}}\Big|_{\mu_t}. \quad (7)$$

Line 3 calculates the so called Kalman gain. This gain is used to update the predicted state based on the measurement signal. H_t is the Jacobian of the measurement model, defined as

$$H_t = \frac{\partial h(x_t)}{\partial x_t}\Big|_{\overline{\mu}_t}. \quad (8)$$

The predicted state and its covariance matrix are updated in lines $4-5$.

The EKF is a suboptimal solution. It is relatively easy to implement; however, linearization of the system equations may cause problems at the points where the behaviour of the system is highly nonlinear.

Algorithm 1 Extended Kalman filter.

Require: state at time $t-1$: μ_{t-1}, Σ_{t-1},
control signal at time t: u_t,
observation at time t: z_t.
Ensure: state at time t:μ_t, Σ_t.
1: $\overline{\mu}_t = f(\mu_{t-1}, u_t)$
2: $\overline{\Sigma}_t = F_t\, \Sigma_{t-1}\, F_t^T + Q$
3: $K_t = \overline{\Sigma}_t\, H_t^T\, (H_t\, \overline{\Sigma}_t\, H_t^T + R)^{-1}$
4: $\mu_t = \overline{\mu}_t + K_t\,(z_t - h(\overline{\mu}_t))$
5: $\Sigma_t = (I - K_t\, H_t)\, \overline{\Sigma}_t$
6: return μ_t, Σ_t

C. Particle Filter

Particle filters approximate the posterior of the state by a finite and random set of states. Unlike the EKF, a particle filter does not linearize the system equations and therefore is a suitable solution for nonlinear systems. However, its space complexity grows exponentially with respect to the dimension of the state variable. Therefore special modifications are required for efficient implementation.

Particle filtering can be summarized in three main steps:

- **Sampling**: This step generates the next generation of particles from the previous generation. Usually the states of the selected particles are predicted using a motion model.
- **Importance Weighting**: Each particle holds an importance weight, denoting the similarity of the proposal distribution to the target distribution. It is defined as

$$w_t^{(i)} = \frac{p(x_{1:t}^{(i)}|z_{1:t}, u_{1:t}, x_0)}{\pi(x_{1:t}^{(i)}|z_{1:t}, u_{1:t}, x_0)}, \quad (9)$$

  where $p(\cdot)$ is the posterior distribution which is referred to as the target distribution and $\pi(\cdot)$ is the proposal distribution (note that equation (9) is defined for full trajectory estimation and is in the general form; later the Markov assumption will be applied to it). The proposal distribution is a distribution which is thought to be close to the target distribution. Particle weights are updated based on the similarity of the proposal and target distributions. Since there is no direct access to the target distribution, it is important to have a good proposal distribution. Choosing a good proposal distribution is very important for the efficiency of the filter. A good proposal would be one with less uncertainty. The more uncertain the proposal distribution, the more particles are required to represent the distribution. Fig. 2 shows this concept. For the peaked distribution[1] only 20 particles suffice to represent it, while for the other distribution which is not peaked, 100 particles are needed to represent the distribution. In robotic applications, especially with sensors such as laser rangers which provide peaked measurement distributions, by choosing a peaked proposal, space complexity can be reduced.

[1] A peaked distribution is one that when approximated by a Gaussian distribution has relatively small 3σ boundaries. In other words, a peaked distribution is less uncertain.

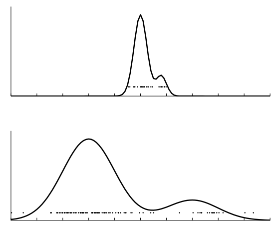

Fig. 2. Generally, for a peaked distribution (top) fewer particles are required than for a highly uncertain distribution (bottom).

- **Resampling**: Usually, after several weight updates, the variance of the weights increases. Most of the particles will have weights close to zero, and the rest will have larger weights. To avoid this particle degeneration, resampling is performed. This process behaves in a similar way to the survival of the fittest in artificial intelligence algorithms. But frequent resampling can also cause degeneracy problems. Calculating the effective sample size, which is a measure of the dispersion of the importance weights, can indicate when to resample. For M particles, this measure is defined as [8]

$$N_{\mathit{eff}} = \frac{1}{\sum_{i=1}^{M}(\hat{w}^{(i)})^2} \tag{10}$$

  where $\hat{w}^{(i)}$ is the normalized weight of the i^{th} particle. N_{eff} varies between 1 (when all particles, except one, have zero weight) and M (when all particles have equal weight of $\frac{1}{M}$). Doucet *et al.* [9] showed that to avoid the degeneracy problem, the best time to resample is when N_{eff} falls below $M/2$.

With this three-step particle filter, there is a key limitation in implementation. Whenever a new observation is made, weights of trajectories should be updated from scratch. Since the length of the trajectory, or the length of the sequence of the poses, is increasing over time, this filtering would fail because of the high computational demand. Doucet *et al.* [9] showed that by choosing the following recursive proposal distribution, it is possible to have a recursive weight-update.

$$\begin{aligned}\pi(x_{1:t}|z_{1:t}, u_{1:t}) = {} & \pi(x_t|x_{1:t-1}, z_{1:t}, u_{1:t}) \\ & \times \pi(x_{1:t-1}|z_{1:t-1}, u_{1:t-1}).\end{aligned} \tag{11}$$

This equation can also be derived from the conditional chain rule.

By placing equation (11) in equation (9) and simplifying the relation, we will have the recursive weight-update [7], [10] (note: the superscript (i) demonstrates the i^{th} particle):

$$\begin{aligned} w_t^{(i)} &= \frac{p(x_{1:t}^{(i)}|z_{1:t}, u_{1:t}, x_0)}{\pi(x_{1:t}^{(i)}|z_{1:t}, u_{1:t}, x_0)} \\ &= \frac{p(x_{1:t}^{(i)}|z_t, z_{1:t-1}, u_{1:t}, x_0)}{\pi(x_t^{(i)}|x_{1:t-1}, z_{1:t}, u_{1:t}, x_0)\, \pi(x_{1:t-1}^{(i)}|z_{1:t-1}, u_{1:t-1}, x_0)} \\ &= \frac{\dfrac{p(z_t|x_{1:t}^{(i)}, z_{1:t-1}, u_{1:t}, x_0)\, p(x_{1:t}^{(i)}|z_{1:t-1}, u_{1:t}, x_0)}{p(z_t|z_{1:t-1}, u_{1:t}, x_0)}}{\pi(x_t^{(i)}|x_{1:t-1}^{(i)}, z_{1:t}, u_{1:t}, x_0)\, \pi(x_{1:t-1}^{(i)}|z_{1:t-1}, u_{1:t-1}, x_0)}. \end{aligned} \tag{12}$$

The term $\dfrac{1}{p(z_t|z_{1:t-1}, u_{1:t}, x_0)}$ has the same value for all particles; therefore, it is treated as a normalizer and replaced by η. Thus, the weight of the i^{th} particle is

$$\begin{aligned} w_t^{(i)} &= \eta\, \frac{p(z_t|x_{1:t}^{(i)}, z_{1:t-1}, u_{1:t}, x_0)\, p(x_{1:t}^{(i)}|z_{1:t-1}, u_{1:t}, x_0)}{\pi(x_t^{(i)}|x_{1:t-1}^{(i)}, z_{1:t}, u_{1:t}, x_0)\, \pi(x_{1:t-1}^{(i)}|z_{1:t-1}, u_{1:t-1}, x_0)} \\ &= \eta\, \frac{p(z_t|x_{1:t}^{(i)}, z_{1:t-1}, x_0)\, p(x_t^{(i)}|x_{1:t-1}^{(i)}, z_{1:t-1}, u_{1:t}, x_0)}{\pi(x_t^{(i)}|x_{1:t-1}^{(i)}, z_{1:t}, u_{1:t}, x_0)} \\ &\quad \times \frac{p(x_{1:t-1}^{(i)}|z_{1:t-1}, u_{1:t-1}, x_0)}{\pi(x_{1:t-1}^{(i)}|z_{1:t-1}, u_{1:t-1}, x_0)} \\ &= \eta\, \frac{p(z_t|x_{1:t}^{(i)}, z_{1:t-1}, x_0)\, p(x_t^{(i)}|x_{1:t-1}^{(i)}, z_{1:t-1}, u_{1:t}, x_0)}{\pi(x_t^{(i)}|x_{1:t-1}^{(i)}, z_{1:t}, u_{1:t}, x_0)}\, w_{t-1}^{(i)} \\ &= \eta\, \frac{p(z_t|x_t^{(i)})\, p(x_t^{(i)}|x_{t-1}^{(i)}, u_t)}{\pi(x_t^{(i)}|x_{t-1}^{(i)}, u_t, z_t)}\, w_{t-1}^{(i)} \end{aligned} \tag{13}$$

In equation 13, in the first probability in the second line, $u_{1:t}$ is dropped, because z_t does not depend on it. Similar concept is applied in the last line. According to equation 13, weights are recursively updated according to the following relation, where $\propto$ shows proportionality of two sides of the equation.

$$w_t^{(i)} \propto \frac{p(z_t|x_t^{(i)})p(x_t^{(i)}|x_{t-1}^{(i)}, u_t)}{\pi(x_t^{(i)}|x_{t-1}^{(i)}, u_t, z_t)}\, w_{t-1}^{(i)}. \tag{14}$$

Notice that in implementation, there is no need to calculate the normalizer, η, directly, since it has a constant value for all particles. However, to consider the effect of the normalizer, a feasible solution is to calculate all weights without the normalizer, and then scale them such that the sum of all weights is one. As a result, in probabilistic robotics, usually equations with normalizers are replaced with two proportional terms without any normalizer, as performed in equation (14).

In equation (14), by choosing the motion model for the proposal, $\pi(\cdot) = p(x_t|x_{t-1}, u_t)$, the weight-update becomes:

$$w_t^{(i)} \propto p(z_t|x_t^{(i)})\, w_{t-1}^{(i)} \tag{15}$$

This equation simply states that weights are updated based on the measurement model, given that the motion model is used as the proposal distribution.

Algorithm 2 summarizes the particle filter. Line 4 performs sampling which is equivalent to the *prediction* in the EKF. The sign $\sim$ in this line shows that $x_t^{(i)}$ is sampled according to the distribution $p(x_t|x_{t-1}^{(i)}, u_t)$. The sampling is based on the motion model. Line 5 updates weights based on the measurement following the relation presented in equation (15). In line 6, the updated particles are added to the particle set of the posterior. If the dispersion of the importance weights, N_{eff}, is less than $M/2$, then resampling proportional to the weights of the particles is performed (line 10). Function $\mathrm{resample}(\cdot)$ does this operation.

Algorithm 2 Particle filter.

Require: Set of particles representing posterior at time $t-1$: S_{t-1},
control signal at time t: u_t,
observation at time t: z_t.
Ensure: Set of particles representing posterior at time t: S_t.

1: $\mathrm{S}_t \leftarrow \emptyset$
2: **for** $i = 1 \rightarrow M$ **do**
3: $\quad < x_{t-1}^{(i)}, w_{t-1}^{(i)} > \leftarrow \mathrm{S}_{t-1}^{(i)}$
4: $\quad$ sample $x_t^{(i)} \sim p(x_t^{(i)} | x_{t-1}^{(i)}, u_t)$
5: $\quad w_t^{(i)} \leftarrow p(z_t | x_t^{(i)})\, w_{t-1}^{(i)}$
6: $\quad \mathrm{S}_t \leftarrow \mathrm{S}_t \cup < x_t^{(i)}, w_t^{(i)} >$
7: **end for**
8: **if** $N_{eff} < \frac{M}{2}$ **then**
9: $\quad$ **for** $i = 1 \rightarrow M$ **do**
10: $\quad\quad \mathrm{resample}(\mathrm{S}_t)$
11: $\quad$ **end for**
12: **end if**
13: return S_t

D. Proposed AUV Localization Method

AUV localization using ASV beacons involves estimating the pose of the vehicle, as shown in Fig. 1. The posterior over the pose of the AUV and the range (with N beacons) is factored as follows:

$$\prod_{n=1}^{N} p(r_n | z_{1:t}, x_{1:t}, x_0)\, p(x_{1:t} | z_{1:t}, u_{1:t}), \tag{16}$$

where r_n demonstrates the range of the n^{th} beacon ($n = 1, ..., N$), $x_{1:t}$ is the pose of the AUV at times 1 to t, $u_{1:t}$ is the control signal, and $z_{1:t}$ is the acoustic measurement of the beacons.

By this factorization, a particle filter is used for calculating the posterior over poses $p(x_{1:t} | z_{1:t}, u_{1:t})$, while calculating the posterior over the range measurement of beacons, $p(r_n | z_{1:t}, x_{1:t}, x_0)$, is performed using EKFs. In other words, the range of each beacon is estimated with an EKF.

To construct the filtering, a particle filter is used to estimate the path of the AUV. Each particle holds the pose of the AUV. The prediction update of the AUV pose is realized through the kinematic model of the AUV. The orientation of the AUV and the control signals are the input to the kinematic model. The importance sampling (discussed in Section II-C) is realized through the acoustic range measurement of the beacon boats. For each of the particles, the range measurement of the beacons is included, where the range of the beacons is estimated using EKFs. Note that each beacon possesses a separate and low-dimension EKF. If there are M particles and N beacons, the i^{th} particle, $i = 1, 2, \ldots M$, is shown as

$$< x_{1:t}^{[i]}, \quad \underbrace{\mu_1^{[i]}, \Sigma_1^{[i]}, \quad \mu_2^{[i]}, \Sigma_2^{[i]}, \quad \ldots \quad \mu_N^{[i]}, \Sigma_N^{[i]}}_{\text{beacons}} >, \tag{17}$$

where $\mu_j^{[i]}$ and $\Sigma_j^{[i]}$ are the mean and covariance of the range of the j^{th} beacon, $j = 1, 2, \ldots, N$. This structure is fundamentally different from other algorithms in which a large multivariate Gaussian is used to estimate range measurements of the beacons. In fact the proposed method can be thought of as a particle filter, composed of many low-dimensional EKFs. Whenever a measurement for the range of a beacon is available, the range is updated by the EKF, as explained in Section II-B.

The EKF is used to estimate the range measurement in order to reduce the range measurement uncertainty. The predicted range measurement is calculated using the known beacon locations m and the estimated AUV pose. The range measurement variance is reduced through the measurement update. The measurement update is realized using the actual acoustic range measurement. The range measurement variance is Q. The beacon locations m are assumed to be known because the GPS fixes are available. $\mu_j^{[i]}$ and $\Sigma_j^{[i]}$ of each beacon are updated using the EKF which is based on the trade off between $\Sigma_j^{[i]}$ and Q. $\Sigma_j^{[i]}$ and Q of all beacons are used as the distribution for importance sampling in order to select more favoured particles that represent both the AUV pose and corresponding range estimates. The corresponding $\mu_j^{[i]}$ and $\Sigma_j^{[i]}$ of all beacons related to each favoured particle are also propagated through importance sampling.

The proposed method has the advantage that it does not need too many particles to approximate the range of the beacons and the pose of the AUV. In other words, with respect to the number of the beacons, N, the complexity of the proposed algorithm is of order $\mathcal{O}(Mlog(N))$, where M is the number of the particles. For a plain EKF implementation, with a large multivariate state vector, the complexity is $\mathcal{O}(N^2)$. The complexity for a simple particle filter is $\mathcal{O}(M^N)$ [7]. Therefore, the proposed algorithm offers computational advantages over straightforward and plain implementation of a particle filter or an EKF. Additionally, the proposed method estimates the full trajectory of the AUV.

III. Simulation Results

Simulation results have been obtained to validate the proposed method. In the simulation, four stationary ASVs are included. The ASV GPS coordinates are known. The global coordinate frame for the localization is defined based on the ASVs.

The kinematic model of the AUV is unknown for this case. First order derivatives are used to predict the AUV motion. As mentioned above, the proposed method is based on the Monte Carlo method. The range information is used for importance sampling for the particle filter.

The location is achieved by fusing the following information:

- ASV GPS coordinates;
- AUV heading;
- AUV forward velocity through water;
- Acoustic range measurement [1].

The localization results are presented in Fig. 3. Notice that the simulation is performed in 2D space, in which the depth information is not taken into account. The short blue and red dashes indicate the heading at each time. The blue dashes represent the average heading of all particles (each blue dot inside the blue circle has a unique heading which is not shown).

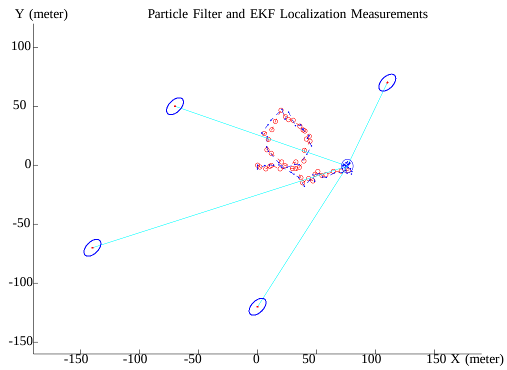

Fig. 3. Trajectory of the AUV. The estimated pose is shown as blue crosses. The ground truth is shown as red circles. The beacons are shown as red dots in blue ellipses. Particles of the last time-step are shown as blue dots. The covariance is shown by the blue circle around the blue dots.

The bounded localization error is shown in Fig. 4. With respect to the number of the beacons, N, the complexity of the proposed algorithm is of order $\mathcal{O}(Mlog(N))$, while for a plain EKF implementation, with a large multivariate state vector, the complexity is $\mathcal{O}(N^2)$. The proposed algorithm offers computational advantages over straightforward and plain implementation of a particle filter or an EKF. In this experiment, 16 particles were used. As the number of the particles increases, the accuracy of the proposed method approaches the accuracy of the EKF. It is also noticeable that with this number of particles ($M = 16$) and four beacons ($N = 4$), the space complexity of the proposed method is less the the complexity of the EKF [7].

IV. Conclusion and Future Work

This paper presents a Monte Carlo method for on-board AUV navigation using acoustic ranges transmitted from multiple autonomous surface vehicles. The approach reduces localization uncertainty while maintaining computational efficiency, which allows for operation in real-time for underwater missions. The concept of reducing the computation demand through combining a PF with multiple EKFs has been described. Simulation results using an AUV and multiple surface vehicles are presented. In the future, specific spatial deployment strategies and observability will be investigated. Future open-water tests with an IVER2 AUV and several ASV will illustrate the efficiency of the method through the reduced error bounded by the navigation, ranging and GPS sensors.

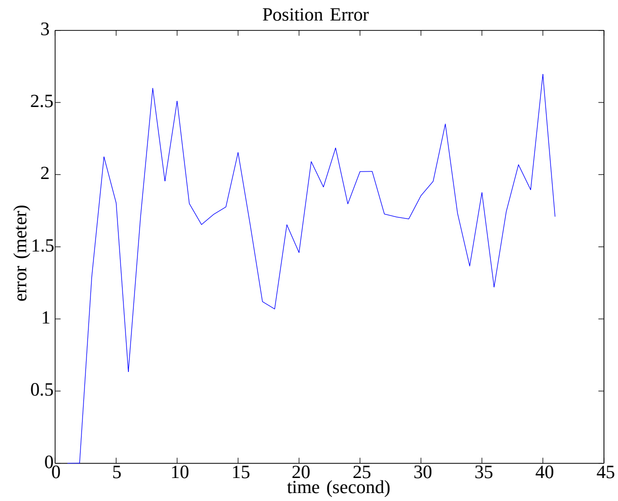

Fig. 4. Bounded localization error.

Acknowledgement

The authors are grateful for the constructive feedback provided by Professor Auke Ijspeert. This research is supported by Natural Sciences and Engineering Research Council of Canada (NSERC) and Canada Foundation for Innovation.

References

[1] L. Paull, S. Saeedi, M. Seto, and H. Li, "Auv navigation and localization: A review," *IEEE JOURNAL OF OCEANIC ENGINEERING*, vol. 39, no. 1, pp. 131–149, 2014.

[2] J. Curcio, J. Leonard, J. Vaganay, A. Patrikalakis, A. Bahr, D. Battle, H. Schmidt, and M. Grund, "Experiments in moving baseline navigation using autonomous surface craft," *Proc. MTS/IEEE OCEANS Conf.*, pp. 730–735, 2005.

[3] A. Bahr and J. J. Leonard, "Cooperative localization for autonomous underwater vehicles," *Proc. Int. Symp. Exp. Robot.*, pp. 1–10, 2006.

[4] M. F. Fallon, G. Papadopoulos, J. J. Leonard, and N. M. Patrikalakis, "Cooperative auv navigation using a single maneuvering surface craft," *Int. J. Robot. Res.*, vol. 29, pp. 1461–1474, 2010.

[5] S. E. Webster, J. M. Walls, L. L. Whitcomb, and R. M. Eustice, "Decentralized extended information filter for single-beacon cooperative acoustic navigation: Theory and experiments," *IEEE TRANSACTIONS ON ROBOTICS*, vol. 29, no. 4, pp. 957–974, 2013.

[6] J. Hudson and M. Seto, "Underway path-planning for an unmanned surface vehicle performing cooperative navigation for UUVs at varying depths," in *Intelligent Robots and Systems (IROS), Proceedings of the IEEE/RSJ International Conference on*, 2014, pp. 2298–2305.

[7] S. Thrun, W. Burgard, and D. Fox, *Probabilistic Robotics*. Cambridge, MA, USA: The MIT press, 2005.

[8] B. Ristic, S. Arulampalm, and N. J. Gordon, *Beyond the Kalman Filter*. Artech House, 2004.

[9] A. Doucet, N. d. Freitas, and N. Gordon, *Sequential Monte Carlo Methods in Practice*. New York: Springer-Verlag, 2001.

[10] G. Grisetti, C. Stachniss, and W. Burgard, "Improved techniques for grid mapping with Rao-Blackwellized particle filters," *IEEE Transactions on Robotics*, vol. 23, pp. 34–46, 2007.