Optimal Design of Rotating Sliding Surface in Sliding Mode Control

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***Abstract*— In sliding mode control, the less is the reaching time to the switching surface, the more robust is the system. In this paper, a systematic control strategy based on the optimal methods has been designed to extract a rotation rule for the sliding surface. Optimality and robustness of the designed controller against disturbance and variations has been shown via simulation results.**

I. INTRODUCTION

Generally, in the design of sliding mode controllers, the following main steps should be done: 1) the determination of a sliding surface that shows the desired stable dynamics and depends on the system type, 2) the extraction of a control rule that guarantees the reaching condition, and 3) the sliding condition. Beginning from the initial conditions, reaching to the desired states, the phase trajectory of a sliding mode system has two distinguished modes [1]. First the trajectories, starting from their initial states off the sliding surface, move towards the sliding surface. This phase is known as the reaching phase and in this mode the system is sensitive to parameter variations. Then after hitting to the sliding surface, the sliding phase starts. In this phase the trajectories are insensitive to parameter variations and disturbances [2]. Therefore, various methods have been suggested to eliminate or reduce the system sensitivity by minimizing or even removing the reaching phase [3].

An improved sliding surface design method for better controller performance is to deploy time-varying linear sliding surfaces instead of constant surfaces of classical methods. The linear sliding surface can be moved by rotating in such a direction that the tracking behavior can be improved.

Hence having the sliding phase and robust behavior from early beginning, the reaching phase can be eliminated or reduced. The initial value of the sliding surface should be designed such that it includes the initial states of the system or if not possible, closer to initial states and the final value should be designed based on system characteristics.

There are several methods that try to reduce reaching phase, even using intelligent neuro-fuzzy methods. In [5] a general method has been introduced, in this method some parameters are designed based on system and design requirements. In [6] a mediating approach has been introduced in which, initial tracking signal is changed to fit to the system initial values. The tracking signal gradually tends to its ideal state to have a perfect sliding phase.

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II. OPTIMAL CONTROL SURFACE DESIGN

In this design approach, the main idea is to use error dynamics to design an optimal sliding surface. Sliding surface start to rotate from the given initial value to stop rotating on reaching to the final value of the sliding surface, but it rotates in such a way that the error dynamics keeps the minimum value based on a given criterion.

Here the design is explained for a second order system of:

$$\ddot{x}(t) = f(x(t)) + u(t)$$

Where $u(t)$ is the control input, $x(t)$ is the (scalar) output of interest, and the dynamics $f(x(t))$ is not exactly known, but estimated as $\hat{f}(x(t))$. The estimation error is assumed to be bounded by some known function $F = F(x, \dot{x})$, i.e.

$$\Delta f(x(t)) = f(x(t)) - \hat{f}(x(t)) \ , \qquad \left|\Delta f(x(t))\right| \le F$$

The output should track a known reference signal $x_r(t)$.

According to design principles [4], using the sign function, the sliding surface and the control signal will be:

$$s(t) = \dot{e}(t) + \lambda e(t)$$

$$u(t) = -\hat{f}(x(t)) - \lambda\dot{e}(t) + \ddot{x}_r(t) - q\,\mathrm{sgn}(s(t))$$

Where

$e(t) = x(t) - x_r(t)$, $\lambda(t)$ is the positive sliding surface slope and q is a positive scalar that is bigger than F to guarantee system stability.

Applying the control signal to the main system, we will have following error dynamic:

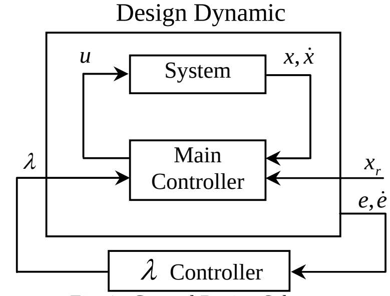


Fig. 1. General Design Scheme

$$\ddot{e}(t) + \lambda\dot{e}(t) = \Delta f(x(t)) - q\,\mathrm{sgn}(\dot{e}(t) + \lambda e(t))$$

This dynamic can be interpreted as mentioned in Fig.1, error dynamics are used to design optimal sliding surface.

Supposing that initial value of λ is such that sliding starts on the sliding surface, and then in the theory we will have:

$$\dot{e}(t) + \lambda e(t) = 0$$

Now the target is to design $\lambda(t)$, such that the following performance index could be minimized:

$$J = \int_{t_0=0}^{t_f=T} (e^2(t) + \dot{e}^2(t))dt$$

In which T is the reaching time to the sliding surface that can be fixed or free. This design can be applied to robotics, welding of metallic surfaces with welding robots where optimal design is important.

Solving the above mentioned equation we will have

$$e(t) = ae^{-\int_0^t \lambda(t)dt}, \quad \dot{e}(t) = -a\lambda(t)e^{-\int_0^t \lambda(t)dt}$$

Where a is the initial value ($a\lambda_m = a\lambda(0) = \dot{e}(0)$). Besides $\lambda(t)$, can have final fixed value of λ_p and initial value of λ_m. Now the new problem can be defined that a continues function of $\lambda(t)$ with given border values should be designed such that the following functional could be minimized:

$$J = \int_0^T (1 + \lambda^2(t))e^{-2\int_0^t \lambda(t)dt}dt$$

Where

$$\lambda(0) = \lambda_m$$

$$\lambda(T) = \lambda_p$$

Intuitively, there should be an answer to the problem because two terms of $1 + \lambda^2(t)$ and $e^{-2\int_0^t \lambda(t)dt}$ have opposite growth by passing the time. In order to simplify the problem, the following variable is introduced:

$$\lambda(t) = \dot{v}(t) \Rightarrow \int_0^t \lambda(t)dt = v(t) + c$$

Hence the functional is:

$$J = \int_0^T (1 + \dot{v}^2(t))e^{-2(v(t)+c)}dt$$

Considering $g = (1 + \dot{v}^2(t))e^{-2(v(t)+c)}$ and using famous Euler equation for optimization [7], $\frac{\partial g}{\partial x} - \frac{d}{dt}(\frac{\partial g}{\partial \dot{x}}) = 0$, we will have the following equation:

$$\ddot{v}(t) = \dot{v}^2(t) - 1$$

Or equivalently:

$$\dot{\lambda}(t) = \lambda^2(t) - 1$$

And this equation results in:

$$\lambda(t) = \frac{\alpha + e^{2t}}{\alpha - e^{2t}}$$

Where by applying initial value, α is:

$$\alpha = \frac{\lambda_m + 1}{\lambda_m - 1}$$

and by applying final value, T or reaching time is:

$$T = \frac{1}{2} Ln \frac{(\lambda_p - 1)(\lambda_m + 1)}{(\lambda_p + 1)(\lambda_m - 1)}$$

Supposing $\lambda_m = 2$ and $\lambda_p = 20$, in Fig.2, $\lambda(t)$ has been depicted, (α is 3 and T is 0.4993). The only important note in using this method is that due to discontinuity of $\lambda(t)$ in its root point, the design of λ_p should be such that before $\lambda(t)$ reaches to the root point, it should have reached to the λ_p and hence avoiding functional problem. In Fig.2, the design of λ_p is such that root point is after reaching time (T) and it is not depicted in Fig.2, because $\lambda(t)$ has already reached to its final value.

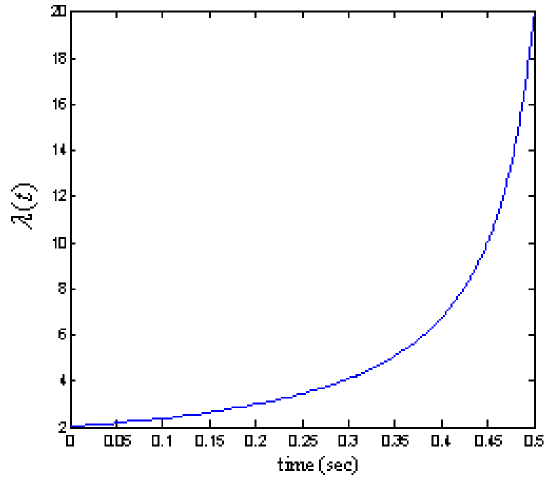


Fig. 2. diagram of $\lambda(t)$

III. SIMULATION

In order to simulate the control performance, the following benchmark system has been used [4]

$$\ddot{x}(t) = -a(t)\dot{x}^2 \cos(3x(t)) + u(t)$$

Where $a(t)$ is considered to be unknown but verifies:

$$1 \le a(t) \le 2$$

In the simulations $a(t)$ is:

$$a(t) = |\sin(t)| + 1$$

$\lambda_m = 2$ that fits to the initial values of the system, $\lambda_p = 20$ that is design criterion.

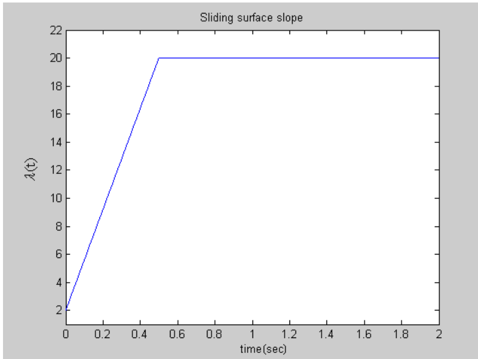
Sampling rate is 1K and T as the reaching time to the final goal sliding surface ($\lambda_p = 20$) is 0.499 second.

Fig.3 shows the system states and $\lambda(t)$ where there is a linear rotation in the surface:

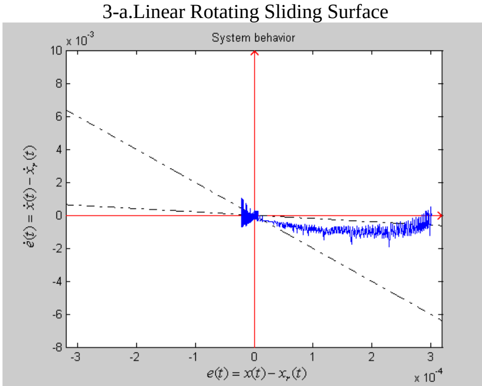
Performance index in this case is:

$$J = 2.5230e^{-4}$$

Fig.4 shows the system states and $\lambda(t)$ where there is rotation in the surface with another non optimal rule, here the following rule is used [5]:

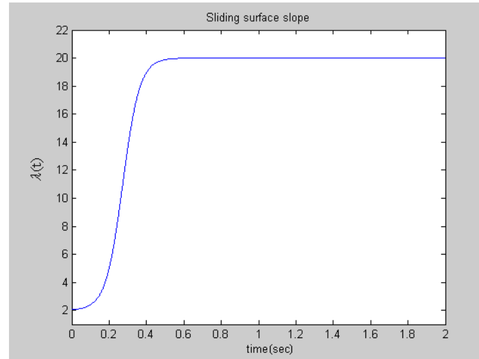


3-a.Linear Rotating Sliding Surface

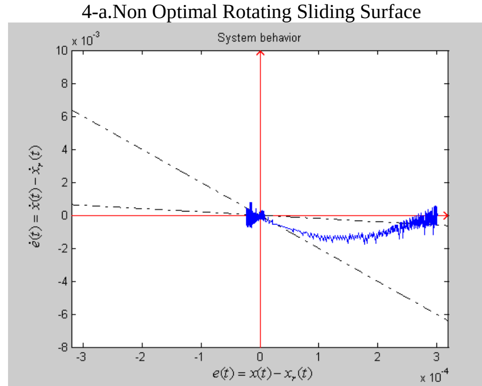


3-b.Linear Rotating Sliding Surface system

Fig.3. Linear Rotating Sliding Surface



4-a.Non Optimal Rotating Sliding Surface



4-b. Non Optimal Rotating Sliding system

Fig.4. Non Optimal Rotating Surface

$$\lambda(t) = \frac{\lambda_p - \lambda_m}{1 + e^{(-at+b)}} + \lambda_m$$

where a and b are used based on experiment.
Performance index in this case is:

$J = 3.0634e^{-4}$

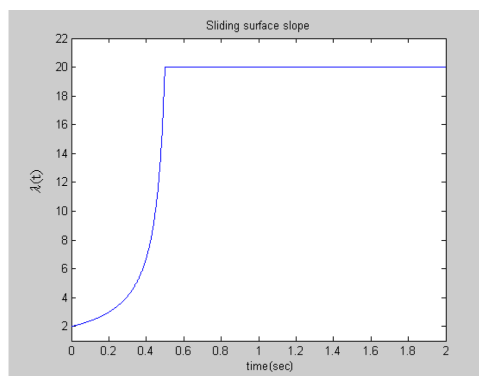
Finally, Fig.5 shows system states and $\lambda(t)$ with the proposed optimal solution.
Performance index in this case is:

$J = 2.0498e^{-4}$

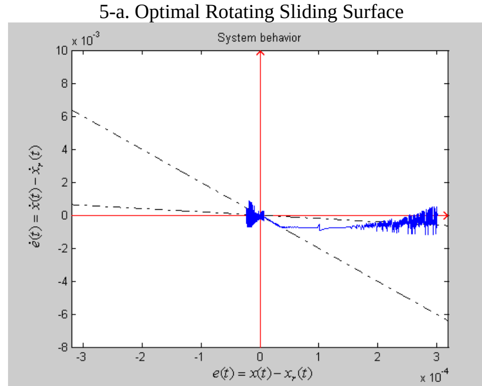
As it is clear, in the case of optimal sliding surface, performance index is less than the other two cases and this proves its advantage via simulation.

IV. Conclusion

This paper was focused on optimal rotating sliding mode control surface design. Simulation has been done in presence of disturbance, and in comparison with other methods, the results of the rotating surface method confirm both stability and optimality of the proposed control method.



5-a. Optimal Rotating Sliding Surface



5-b. Optimal Rotating Sliding system

Fig.5. Optimal Rotating Surface

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