A Neural Network-based Multiple Robot Simultaneous Localization and Mapping

Sajad Saeedi†, Liam Paull†, Michael Trentini* and Howard Li†

Abstract—In this paper, a decentralized platform for Simultaneous Localization and Mapping (SLAM) with multiple robots is developed. Each robot performs single robot view-based SLAM using an Extended Kalman Filter (EKF) to fuse data from two encoders and a laser ranger. To extend this approach to multiple robot SLAM, a novel occupancy grid map fusion algorithm is proposed. Map fusion is achieved through a multi-step process that includes image pre-processing, map learning (clustering) using neural networks, relative orientation extraction using norm histogram cross correlation and a Radon transform, relative translation extraction using matching norm vectors and then verification of the results.

The proposed map learning method is a process based on the Self Organizing Map (SOM). In the learning phase, the obstacles of the map are learned by clustering the occupied cells of the map. The learning is an unsupervised process which can be done on-the-fly without any need to have output training patterns. The clusters represent the spatial form of the map and make further analyses of the map easier and faster. Also, clusters can be interpreted as features extracted from the occupancy grid map so the map fusion problem becomes a task of matching features. Results of the experiments from tests performed on a real environment with multiple robots prove the effectiveness of the proposed solution.

I. Introduction

Robotics systems and missions are becoming more complex. As a result, it is important for agents and sub-systems to work in parallel and share information to achieve difficult tasks more efficiently. One such task is SLAM or Simultaneous Localization and Mapping. In SLAM, a robot is placed in an a priori unknown environment and tries to build a map of the environment and also situate itself within the map simultaneously [1]. SLAM is useful in a variety of missions, such as: rescue operations, fire fighting, underwater and space exploration and surveillance among others. The task is very challenging because it involves processing large amounts of data from different heterogenous sensors such as cameras [2], inertial measurement units (IMUs), and laser rangers [3]. Moreover, the task must be completed in real-time with restricted onboard processing capabilities. Different variations of single robot SLAM have been proposed in the literature to account for variations in data fusion algorithms and sensing devices. Approaches include competition-based neural networks [4], Sparse Extended Information Filter (SEIF) [5], and Extended Kalman Filtering (EKF) [6] among others [7], [8].

More recently, researchers have focused on developing distributed systems to achieve SLAM. In multiple robot SLAM, many robots are used to map an unknown environment without any prior knowledge of their locations or the locations of other members of the team. Using multiple robots or agents can have many advantages, for example: the total time required to explore an environment can be greatly reduced, system performance can be improved because more information is being provided by different sources, and a distributed system is more robust to failures. However, multiple robot SLAM has the added difficulty that the maps generated from each robot must be merged to make a global map. Each agent must achieve its own localization and mapping goals while also contributing to global goals and cooperating with others. Map merging is made more difficult by the uncertainty in the individual maps and also the fact that the robots can not be assumed to know their relative poses.

Different approaches exist in the literature for overcoming these challenges. A feature-based multiple robot SLAM algorithm is proposed in [9], which uses an information filter (IF). In [10], a fuzzy logic based multiple robot SLAM algorithm is proposed where fuzzy sets are used to represent the uncertainty of the position and the fuzzy intersection operation is used to fuse data. The proposed method in [11] uses a particle filter (PF), with the limiting assumption that the robots will meet each other at a point. At the meeting point, relative positions and orientation of the robots are extracted and then relative initial positions are determined by backwards calculation. The occupancy map merging solution proposed in [12] is an effective solution for multiple robot SLAM. In this method, a similarity index for the maps is developed based on the map-distance. Then, similar patterns in two maps are found based on a random walk algorithm. The random walk may be highly time consuming and insufficient distinctive patterns in the maps may cause failure.

All of the previously published methods either have invalid assumptions or are so computationally intensive that the robots will have to move very slowly for real-time operation. A more efficient computational approach will result in the robots being able to move more quickly through the environment and, consequently, achieving the task in a shorter time.

Neural network algorithms are powerful artificial intelligence solutions which can be applied to any problem involving learning or training. In multiple robot SLAM there is the need to learn a map. After learning a map, any integration with other maps will be much faster and simpler.
due to the reduced dimensionality. Many different neural network architectures exist. Feedforward networks are popular due to their simple structure, however, the training methods require supervision, or input and output patterns, to train the network. Since the map is unknown, these input-output pairs are not available. Kohonen networks, or self organizing maps (SOM), which have a recurrent structure, use unsupervised training to reduce the dimensionality of input data. This means that only input patterns are required to train the network. This type of network is a good candidate to solve the map fusion problem because the weights and parameters of the network are calculated without any need to specify output patterns.

There is no known literature that applies neural networks to the multiple robot SLAM problem.

In this paper a new method of map fusion for multiple robot SLAM is introduced. A new algorithm is proposed which is based on neural network theory. In this new approach, the map is learned by clustering, which reduces complexity and increases speed and accuracy. The maps are fused into a global map using relative transformation matrices determined using the clustered points.

To summarize, the contributions of this research include:
- A high level map segmentation algorithm for occupancy grid map preprocessing.
- Application of SOM to cluster (learn) the preprocessed map.
- Inclusion of map uncertainty in the learning phase.
- Estimation of the relative transformation matrix of two maps using the cluster points.
- The use of surface norms to associate cluster points from two different maps.

The rest of the paper is organized as follows: Section II presents the proposed methods, Section III contains results of two experiments, and Section IV makes some general conclusions and discusses future work.

II. PROPOSED METHODS

Two important issues with multiple robot SLAM are finding the relative initial poses of the robots and coping with the uncertainty of the maps of the robots. Since the maps are considered to be occupancy grid maps, the fusion algorithm should take into account the associated uncertainties; More emphasis should be given to the cells with lower uncertainties of occupancy. In this research, the map learning process is performed by clustering the map into specific clusters using unsupervised Kohonen networks. The clusters represent the main characteristics and features of the map and make further map processing easier and faster.

The main goal of the proposed method is to find the relative transformation between maps as quickly and reliably as possible, while considering the uncertainties. An overview of the method is shown in Fig. 1. The inputs of the algorithm are the two occupancy maps and the output is their relative transformation.

A. Map Segmentation

The first operation performed on the maps is segmentation. The segmentation identifies discontinuous segments of obstacles located far enough from each other that they should be considered as separate. This is necessary so that the SOM algorithm can be run on each contiguous segment of obstacle in the map. Otherwise, the SOM will produce clusters that are not close to obstacles but could be the midpoint between two separate obstacles. Let $M$ be the set of all unclustered obstacle points in the occupancy grid map.

To begin, a random point, $p_0 = (x_0, y_0)$, is selected from $M$, and is added labeled as a start point. The set of all start points is maintained in the set $S$, which at first contains only $p_0$. The integer $n$ is the cluster number and is initialized to a value of 1. The set that contains all points in cluster $n$ is $C^n$. The algorithm continues for each cluster as long as there are start points remaining in $S$ to be processed. For each start point, $p_s = (x_s, y_s)$, the set of neighboring points, $C_s$, is determined by:

$$C_s = \{ p = (x, y) | (x - x_s)^2 + (y - y_s)^2 < r^2 \}, \quad (1)$$

Next, the set of potential new start points, $S_s$, is calculated. A ring of width 1 is first searched as given by:

$$S_s = \{ p = (x, y) | r^2 < (x - x_s)^2 + (y - y_s)^2 < (r + 1)^2 \}. \quad (2)$$

If this search yields no results, then the width of the ring is increased to $\Delta r$ as given by:

$$S_s = \{ p = (x, y) | r^2 < (x - x_s)^2 + (y - y_s)^2 < (r + \Delta r)^2 \}. \quad (3)$$

The reason for doing this process in two steps is to try to limit the number of potential next start points as much as possible to increase the speed of the algorithm. The set of next start points is updated to include the members of $S_s$ that are not already in the cluster $C^n$, and the current start point, $p_s$ is removed. The cluster, $C^n$, is updated to include the values of $C_s$ and $S_s$.

Once $S$ is empty, the cluster is finished. The points in the cluster are removed from $M$ and $n$ is increased. The
process continues until $M$ is empty. The detailed algorithm is presented in Algorithm 1, and Fig. 2 shows a graphical representation of the process.

**Algorithm 1 Map Segmentation**

**Input:** Set of occupied cells from occupancy grid map ($M_{occ}$)
- search radius ($r$)
- search radius increase step ($\Delta r$)

**Output:** $n$ set of clusters: $C_i$, $i = 1, \ldots, n$

1. $n \leftarrow 0$
2. $M \leftarrow M_{occ}$
3. While $M \neq \emptyset$
   1. $n \leftarrow n + 1$
   2. $p_0 \leftarrow$ random element of $M$.
   3. $S \leftarrow \{p_0\}$
   4. While $S \neq \emptyset$
      1. $p_s \leftarrow (x_s, y_s) \leftarrow$ random element of $S$
      2. $C_s \leftarrow \{p = (x, y) | (x - x_s)^2 + (y - y_s)^2 < r^2\}$
      3. $S_s \leftarrow \{p = (x, y) | r^2 < (x - x_s)^2 + (y - y_s)^2 < (r + \Delta r)^2\}$
      4. If $S_s = \emptyset$ then
         1. $S \leftarrow S \cup (S_s - C^n)$
         2. $C^n \leftarrow C^n \cup C_s \cup S_s$
      End if
   End while
   5. $M \leftarrow M - C^n$
End while

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**B. Clustering with SOM**

Once the map segmentation is complete, the clustering is performed on each segment. The clusters, which are generated by applying SOM, will be used to fuse the two maps (refer to Fig. 1). The purpose of performing this clustering is to decrease the time required for map fusing. The clusters preserve the original information or shape of the obstacles or features from the map but represents them in a much more compact form.

Fig. 3 shows the simple structure of the network. In this case, the data is a two dimensional position of an occupied cell in the occupancy grid map. This should not be confused with the locations of the neurons in the solution space, which are also two dimensional locations. In the training phase, the location of each occupied cell is presented as the input of the network. The network finds the best matching or winning neuron to be the one whose weights, or data, are closest to that input position. The weights corresponding to that neuron and the neurons that are in the neighborhood in the solution space are updated to better represent that input pattern. Once the algorithm is run for several iterations, the final weights are the clusters that represent the data.

The number of neurons or clusters, $n$, is an important design parameter. Its selection is dependant on the size of the map, and it is noted that the choice of $n$ has a significant effect on the quality of the results.

It is emphasized that the training process and then subsequent map fusion are fast enough that they can be implemented onboard a robotic platform.

1) **Map Uncertainty**: A major advantage of the occupancy grid map representation is the treatment of uncertainty. Values in an occupancy grid map are float values in the range $(-1, 1)$ where negative values indicate belief of an obstacle, and positive values indicate belief that the cell is free. At first, all cells are initialized to 0, indicating that the state of the cell is unknown. As sensor data is processed, the values in the map are adjusted. For example, if a cell is detected as being occupied by many scans, it will have a value closer to $-1$ than if it had only been detected as occupied a small number of times.

In order to account for this uncertainty, input patterns with higher certainty of being obstacles are presented more frequently as the input in the training of the SOM.

Here, each time a cell, $C$, is detected as occupied, the value is decreased by some tunable parameter $\Delta$ to a minimum of $-1$. Alternately, each time $C$ is detected as free, its value
and are equidistant from two clusters. The relative orientation are unit vectors that are perpendicular to the cluster surface (dotted line). The cluster surface norms surface generated by connecting adjacent clusters is termed C. Relative Orientation

involves extracting the relative orientation and translation formation between the maps can be found efficiently. This presents is determined by the relation:

$$ f(C) = \text{round}\left(\frac{|C|}{\Delta}\right), $$

(4)

For example, if $\Delta = 0.1$, a cell with a value of $-0.4$ should appear as the input to the SOM four times more often than a cell with a value of $-0.1$.

Once the clustering has been performed, the relative transformation between the maps can be found efficiently. This involves extracting the relative orientation and translation separately as will be discussed in the next sections.

C. Relative Orientation

Fig. 4 shows an illustration of four cluster points. The surface generated by connecting adjacent clusters is termed the cluster surface (dotted line). The cluster surface norms are unit vectors that are perpendicular to the cluster surface and are equidistant from two clusters. The relative orientation between the two maps is determined by performing a 360° histogram on the directions of the cluster surface norms, and then matching the histograms of the two maps. Some sample histograms are shown in the experimental results section. This is similar to the circular cross correlation of histogram norms that is presented in [6] for aligning two consecutive local maps. However, because the number of data points has been effectively reduced through clustering, the size of the buckets in the histogram must be relatively large. As a result, it is not possible to get sufficiently accurate results. A tuning method based on the Radon transform is used to find a more accurate value for the relative orientation [13].

1) Tuning with the Radon Transform: The Radon transform is the projection of the image intensity along a radial line oriented at a specific angle [14]. The Radon image is generated by computing a Radon transform and varying the Radon angle, $\theta$, of the radial line onto which the original image is being projected. For a given map, $m(x,y)$, the Radon transform along the radial line of angle $\theta$ is defined as:

$$ r_{\theta}(x') = \int_{-\infty}^{\infty} m(x' \cos \theta - y' \sin \theta, x' \sin \theta + y' \cos \theta) dy', $$

(5)

where

$$ \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}. $$

(6)

The Radon image generated by computing a Radon transform is a collection of all possible projections for a larger set of angles. In a Radon image, the horizontal axis represents all possible angles and the vertical axis is the Radon transform for individual angles. One of the characteristics of the Radon image is that the peak points occur when the Radon angle, $\theta$, aligns with straight line segments that occur in the image. As a result, it is possible to resolve the exact rotation by looking for peaks in the Radon images of both maps that are close to the approximate angle previously computed using the histogram method. The difference of the peak angles is the tuning angle that should be added to the original approximate angle as determined by:

$$ \alpha = \alpha_{app} + \delta_{tuning}, $$

(7)

$$ \delta_{tuning} = \arg \min_{\theta} (R_{\theta=0:180}(m_1)) - \arg \min_{\theta} (R_{\theta=0:180}(T_{\omega}(m_2))), $$

(8)

where $\alpha$ is the tuned rotation angle, $\alpha_{app}$ is the result of the cross correlation and $R_{\theta=0:180}(map)$ is the Radon image of the input map over the specified domain of angles ($\theta$). The operation $T_{\omega}(map)$ is a rotation of the input, map, by $\omega$ and $m_1 = map_1$ and $m_2 = map_2$.

Once the tuned relative orientation has been found, it is applied so that the relative translation can be found as will be discussed presently.

D. Relative Translation

Referring again to Fig. 4, the cluster point norms are unit vectors that originate from the clusters at an angle that is the average of angles of the two adjacent cluster surface norms. For cluster points that are at the end of the cluster surface, the point norms are parallel to the surface norms. These point norms are used to find the relative translation between the two maps using an iterative approach similar to the Iterative Closest Point (ICP) algorithm [15].

Let $M_1 = \{m_1^1...m_1^K\}$, and $M_2 = \{m_2^1...m_2^K\}$ be the sets of cluster points for $map_1$ and $map_2$ respectively after the relative rotation has been applied. The associated sets of point norms are $N_1 = \{n_1^1...n_1^K\}$, and $N_2 = \{n_2^1...n_2^K\}$ (the norm of $m_k^i$ is $n_k^i$ for all $k$. Similar for $M_2$). Without loss of generality, $M_2$ is being translated to $M_1$.

For each point $m_k^i, k = 1..K$ in $M_2$, a new set, $P_k$ ($P_k \subseteq M_1$) is built that contains the points in $M_1$ that have norms within a matching angle threshold, $\epsilon$ of $n_k^i$ (the point norm of $m_k^i$). If this set $P_k$ is non-empty, then the point in $P_k$ that has the smallest Euclidean distance from $m_k^i$ is chosen as the winner and a correspondence is made. These correspondences are maintained in two new sets, $PT_1$ and $PT_2$. If the set $P_k$ is empty, it means that there were no points in $M_2$ with similar norms, or that this part of $map_2$
has no matching part in map_1, so no correspondence is made and the algorithm continues.

Once all of the points in M_2 have been considered, the best translation is found by evaluating the equation:

\[
T = \begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix} = \begin{bmatrix} \frac{1}{L} \left( \sum_{i=1}^{L} PT_{1x}[l] - \sum_{i=1}^{L} PT_{2x}[l] \right) \\ \frac{1}{L} \left( \sum_{i=1}^{L} PT_{1y}[l] - \sum_{i=1}^{L} PT_{2y}[l] \right) \end{bmatrix}
\] (9)

where L is the cardinality of the sets PT_1 and PT_2, and the values PT_{1x}[l] and PT_{1y}[l] correspond to the x and y components at location l of the array. Similarly for PT_2.

Once the translation is found, it is applied to the set M_1 and matching angle threshold is reduced and the algorithm is restarted. The stopping criterion is either a number of iterations or a minimum error threshold is reached between the two cluster sets M_1 and M_2. The method is summarized in Algorithm 2.

Algorithm 2 Translation realization based on the norms of the cluster points

Input: matching angle threshold (\(\epsilon\))
The maximum number of iterations: \(\text{iterations}_{\text{max}}\)
The error threshold: \(J_{\text{thresh}}\)
Sets of cluster points M_1 and M_2 with associated sets of point norms N_1 and N_2
Output: Translation, T

\[
\text{iterations} \leftarrow 0
\]

repeat
\[i = 1;\]
for \(k = 1 \rightarrow K\) do
\[P_k \leftarrow \{\text{All points } p = (x, y) | p = m_1 \in M_1 \text{ whose associated norm } n_1 \text{ satisfies } |n_1 - n_2| < \epsilon\}\]
if \(P_k \neq \emptyset\) then
\[PT_1[i] \leftarrow \text{element of } P_k \text{ that is closest to } m_2^k\]
\[PT_2[i] \leftarrow m_2^k\]
\[i \leftarrow i + 1\]
end if
end for
\[\delta_x \leftarrow \frac{1}{\sum_{i=1}^{L} \left| \sum_{i=1}^{L} PT_{1x}[l] - \sum_{i=1}^{L} PT_{2x}[l] \right|}\]
\[\delta_y \leftarrow \frac{1}{\sum_{i=1}^{L} \left| \sum_{i=1}^{L} PT_{1y}[l] - \sum_{i=1}^{L} PT_{2y}[l] \right|}\]
\[T = \begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix}\]
Each point in M_2 gets shifted by T
\[J = \sum_{i=1}^{L} \left| \sum_{i=1}^{L} PT_{1}[l] - PT_{2}[l] \right|\]
Reduce \(\text{iterations}\) until \(J < J_{\text{thresh}}\) or \(\text{iterations} > \text{iterations}_{\text{max}}\).

It is noted that the selection of the matching angle threshold, \(\epsilon\), is quite important. Values that are too large give inaccurate results and values that are too small do not yield any correspondences. An effective tradeoff is to reduce the value of \(\epsilon\) at every iteration. In this case it was initialized to 25° and reduced gradually until it reached a minimum of 4°.

E. Verification

A final verification process is used to eliminate false matching or to select the best result of a few candidates. The verification method is based on the convergence of the performance index, J as given by:

\[J = \sum_{i=1}^{n} ||p_1(i) - p_2(i)||, \] (10)

where \(n\) is the number of clusters, and \(p_1(i)\) and \(p_2(i)\) are the corresponding cluster points for map_1 and map_2 respectively. If \(J\) converges or falls below a predetermined threshold, then the fusion of the clusters can be considered as valid.

In order to ensure that the entire maps have been accurately fused, the verification index given in (11) is used [12], where the relative orientation and translation have been applied to map_2.

\[V(\text{map}_1, \text{map}_2) = \frac{\text{agr(\text{map}_1, \text{map}_2) \times 100\%}}{\text{agr(\text{map}_1, \text{map}_2) + dis(\text{map}_1, \text{map}_2)}}. \] (11)

The two components \(\text{agr(m}_1, m_2)\) and \(\text{dis(m}_1, m_2)\) are defined as:

\[\text{agr(m}_1, m_2) = \sharp\{p = (x, y) | \text{map}_1(p) = \text{map}_2(p)\}\],
\[\text{dis(m}_1, m_2) = \sharp\{p = (x, y) | \text{map}_1(p) \neq \text{map}_2(p)\}\], (12)

where the operator \(\sharp\) over a given set returns the cardinality of the set. The function \(\text{agr}()\), the agreement index, is the number of known cells with equal status in both maps (either both occupied or both free). The function \(\text{dis}()\), the disagreement index, is the number of cells which are known in at least one map and have unequal status.

The verification index \(V(\text{map}_1, \text{map}_2)\) gives a measure of how similar the two maps are. A similarity index of 100% would mean that the two maps are identical.

III. Experiment

Two experiments are performed on real-world indoor environments. Two differential-wheeled robots built by CoroWare, Inc. are used and each is equipped with two high speed Phidget Encoders and a UBG-05LN laser ranger from Hokuyo, as shown in Fig. 5. The laser ranger sensor has a range of 4500 millimeters with an angle resolution of 0.36 degrees resulting in 513 points for each scan. Data from these sensors are fused with an EKF [16]. The encoder is used...
to provide an estimation of the control signal. Consecutive scans of the laser ranger are preprocessed by an Iterative Closest Point (ICP) algorithm [17] to update the pose. The position of the robot and the map of the environment are shared with other robots. Maps are in the occupancy grid format with the size of 400 × 400 cells. Gray cells are unknown cells, white cells are free cells, and black cells are occupied cells.

### A. Experiment 1: Simple Test Environment

Fig. 6 shows the approximate floor plan and dimensions of an indoor environment used for the experiment. This room has been surveyed by two robots from different initial positions which are assumed to be unknown.

Fig. 7-a shows the local map provided by robot 1, and Fig. 8-a shows the local map provided by robot 2.

After applying map segmentation to both maps, Fig. 7-b and Fig. 8-b show the different map segments in different colours. In this case the larger segments from each of the maps are matched. Runs of the map segmentation on a 2.66 GHz Intel based computer yielded an average run-time of about 2 seconds. The results of the clustering using SOM are shown in Fig. 7-c and Fig. 8-c. Clusters tend to keep the spatial characteristics of the map, so any transformation which can fuse the clusters is a good estimate for the fusion of the maps. The training of the SOM is the most time consuming step of the process. For 40 cluster points, the average time for training is about 20 seconds and the result converges after 5 iterations. Fig. 7-d and Fig. 8-d depict the normals histogram of the clustered points. Applying circular correlation to the orientation histograms, gives the relative orientation of 57.5 degrees. Fig. 9-a shows the clusters after the rotation by the cross correlation. Fig. 10-a and Fig. 11-a show the Radon images for map 1 and map 2 respectively after applying the approximate transformation. As Fig. 10-b and Fig. 11-b show, the peak point of the Radon image for map 1 is at 91° while for the rotated map 2 it is 93°. The difference of 2° is the tuning angle. Fig. 9-b shows the clusters after rotation tuning by the Radon transform. Using the proposed iterative approach, translation is found. In Fig. 9-c, the translated clusters are shown. The translation vector is calculated as (49.3, 59.8). Finally, Fig. 9-d shows both maps fused using the extracted rotation and translations. Fig 12 shows the performance index defined in (10). The error converges after 15 iterations. The algorithm may be stopped at this point.

In total, the algorithm takes about 30 seconds to run. This is about one third of the time that the algorithm presented in [13] required to run on the identical environment producing similar results. The verification index, $V(map_1, map_2)$ defined in (11) is 95%, which shows that the two maps are indeed quite well matched.
B. Experiment 2: More Complex Environment

To demonstrate the effectiveness of the proposed methods, another experiment is performed in a larger environment with the approximate size of $17 \times 10$ meters. Fig. 13 shows the approximate floor plan of the test environment with the paths of the robots.

As in the previous experiment, each robot generates a local occupancy grid map of the environment. At certain specified time intervals, the robots share their local maps with each other. Fig. 14-a and Fig. 14-b show the two local maps generated by the robots. Note that the left part of the test environment was out of the range of the laser range scanners. The two maps, map_1 and map_2, are provided by robot_1 and robot_2 respectively. The proposed map fusion algorithm is assumed to be done on robot_1 with map_2 being integrated into map_1. Fig. 14-c shows the result of the map fusion of map_2 into map_1 using the proposed method. The transformation vector for this case is $T_{x,y,\theta} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}^T = \begin{bmatrix} -1.2 \\ -32.9 \\ 26^\circ \end{bmatrix}^T$. In this case, the entire algorithm required about 65 seconds to run, which compares with over 2 minutes for the algorithm presented in [13]. The verification index, $V(map_1, map_2)$ defined in (11) is 95%.
of unknown environments, which is useful in a variety of different robotics applications.

In the future, an adaptive method of determining the appropriate number of clusters will be investigated. Also, incorporating information gained if the robots do happen to see each other in the environment can yield better results because it provides a way for the robots to quickly easily find their relative poses.

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